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Compton scattering and electron-atom scattering in an elliptically polarized laser field of relativistic radiation power

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Abstract. Presently available laser sources can yield powers for which the ponderomotive energy of an electrons U_p can be equal to or even larger than the rest energy mc^2 of an electron. Therefore it has become of interest to consider fundamental radiation-induced or assisted processes in such powerful laser fields. In the present work we consider laser-induced Compton scattering and laser-assisted electron atom scattering in such fields, assuming that the laser beam has arbitrary elliptic polarization. We investigate in detail the angular and polarisation dependence of the differential cross-sections of the two laser-induced or laser-assisted nonlinear processes as a function of the order N of absorbed or emitted laser photons ω. The present work is a generalization of our previous analysis of Compton scattering and electron-atom scattering in a linearly polarized laser field [Phys. Rev. A **65**, 022712 and 033408 (2002)].

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1 Compton scattering

1.1 Introduction

The investigation of Compton scattering has a long history. Surveys on early work about laser-induced Compton scattering can be found in the review of Eberly [1]. Classical Thomson scattering in a powerful, plane wave radiation field was analyzed analytically and numerically by several authors [2]. With the advent of very powerful laser sources, the predicted energy and momentum shifts in a laser field became observable and were analyzed carefully by Meyerhofer and co-workers [3]. Their experimental findings gave rise to renewed interest in the investigation of relativistic Thomson scattering in a very powerful laser field. The quantum mechanical Compton process was reinvestigated by Narozhnyĭ and Fofanov [4], considering a powerful laser pulse of circular polarization. On the whole, however, very little numerical work was done.

It is the purpose of the present work to generalize our previous investigation [5,6] and reanalyze laser-induced Compton scattering in a very powerful radiation field of arbitrary polarizations in which the ponderomotive energy U_p of the electron in the field is of the order of magnitude of the electron's rest mass, $U_\mathrm{p} \simeq mc^2$, or even larger. For elliptic laser polarization, our present calculations will lead to a new form of generalized Bessel functions. We shall use units $\hbar = c = 1$ throughout this work.

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1.2 The nonlinear Compton formula

The solution of the Dirac equation for an electron moving in an arbitrary electromagnetic plane wave field was derived by Volkov [7] and later also by Denisov and Fedorov [8]. Using a Volkov solution $\psi_{p_\mathrm{i}}(x)$ for an ingoing electron of four momentum p_i and a similar solution $\psi_{p_\mathrm{f}}(x)$ for the scattered electron of four-momentum $p_{\mathrm{f}\mu}$, we can write down the transition matrix element T_{fi} of laser-induced Compton scattering by a Dirac particle, using the interaction Hamiltonian of this process, given by

$$H_\mathrm{int} = e\overline{\psi}_{p_\mathrm{f}}(x)\gamma^\lambda\psi_{p_\mathrm{i}}(x)A'_\lambda(x), \tag{1}$$

where the effective vector potential $A'_\lambda(x)$ for the scattered photon of frequency ω' and wave vector $\mathbf{k}'$, evaluated from the quantized field operator $\hat{A}'_\lambda$, reads

$$\left\langle 1_{k'}\left|\hat{A}'_\lambda\right|0_{k'}\right\rangle = \sqrt{\frac{2\pi}{V\omega'}}\epsilon'_\lambda \mathrm{e}^{\mathrm{i}k'x}. \tag{2}$$

For a powerful laser field of arbitrary elliptic polarisation, described by a monochromatic plane wave with the vector potential

$$A_\mu(kx) = A_0(\epsilon_{1\mu}\cos(\delta)\cos(kx) + \epsilon_{2\mu}\sin(\delta)\sin(kx))$$
$$\epsilon_1 = (0, \boldsymbol{\epsilon}_1), \qquad \epsilon_2 = (0, \boldsymbol{\epsilon}_2)$$
$$\epsilon_1^2 = \epsilon_2^2 = -\boldsymbol{\epsilon}_1^2 = -\boldsymbol{\epsilon}_2^2 = -1,$$
$$\epsilon_1\epsilon_2 = \boldsymbol{\epsilon}_1 \cdot \boldsymbol{\epsilon}_2 = 0, \tag{3}$$

we can perform the Fourier decomposition of T_{fi} and obtain $T_{fi} = \sum_{N=-\infty}^{+\infty} T_N$ where the transition matrix elements T_N of the different nonlinear processes with the absorption of N laser photons ω have to be evaluated explicitly. This calculation is straightforward and yields

$$T_N = -\mathrm{i}e(2\pi)^4\sqrt{\frac{2\pi}{V\omega'}}\frac{m}{V\sqrt{E_\mathrm{f}E_\mathrm{i}}} \times \delta^4(\bar{p}_\mathrm{f} - \bar{p}_\mathrm{i} + k' - Nk)M_N \tag{4}$$

where the laser-dressed four-momenta are defined by

$$\bar{p} = p + d\ n, \quad \mu = \frac{|eA_0|}{m}$$
$$n = \frac{k}{|k_0|}, \qquad d = \frac{m^2\mu^2}{4np} \tag{5}$$

and the matrix elements M_N were explicitly written down in our recent work [6]. These matrix elements can be evaluated numerically and do not concern us here.

Consequently, the nonlinear cross-sections of the order N of laser-induced Compton scattering is given by

$$\frac{\mathrm{d}\sigma_N}{\mathrm{d}\Omega_{\vec{k}'}} = r_0^2\left(\frac{\omega'}{\omega}\right)\left(\frac{m}{E_\mathrm{f}}\right)\frac{|M_N|^2}{\mu^2(1 - \mathbf{n}\cdot\boldsymbol{\beta}_i)}$$

where $r_0 = e^2/m$ is the classical electron radius. From the energy and momentum conservation relations, expressed by the δ-functions in the T-matrix elements (Eq. (4)), the frequencies of scattered radiation are found to be

$$\omega' = \frac{N\omega(E_i - \mathbf{p}_i\cdot\mathbf{n})}{E_i - \mathbf{p}_i\cdot\mathbf{n}' + (N\omega + d)(1 - \mathbf{n}\cdot\mathbf{n}')}. \tag{6}$$

1.3 Numerical examples

We present cross-sections of Compton scattering for the radiation field intensity $I = 10^{21}$ Wcm^{-2} of a Ti:sapphire laser of frequency $\omega = 1.54$ eV. The laser beam is counter-propagating to a beam of electrons of momentum $\mathbf{p}_i$ and of small kinetic energy $E_\mathrm{kin} = 10$ eV. We present data for $N = 10^3$ absorbed laser photons. The scattered light is taken to have linear polarization, described by its unit vector $\boldsymbol{\epsilon}'$. We define the scattering plane by the vectors $\mathbf{p}_i$ and the unit vector $\boldsymbol{\epsilon}_1$ of a linearly polarized laser field. We consider that $\boldsymbol{\epsilon}'$ is either in this plane or perpendicular to it. These two scattering configurations will be denoted by the symbols || and $\perp$, respectively.

In Figure 1 we show in units of r_0^2 the cross-section for linear laser polarization ($\delta = 0°$) for the emission of Compton light having either its polarization $\|$ to the scattering plane (left panel) or $\perp$ to it. Looking at the ordinate scales of these two figures, it is quite obvious that the $\|$ polarization of the Compton light dominates over the $\perp$ one, but in both cases the Compton radiation is emitted predominantly into very small scattering angles close to the direction of propagation of the laser beam.

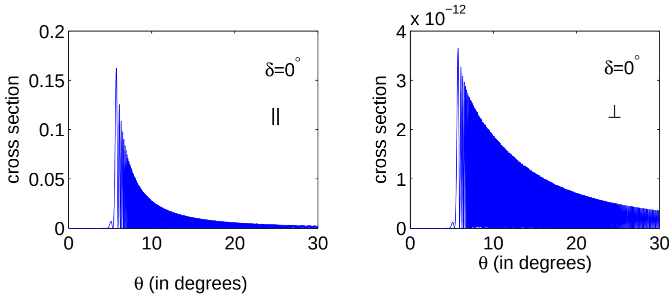

Fig. 1. Cross-sections of Compton scattering in units of r_0^2 in a linearly polarized laser field ($\delta = 0$) for $I = 10^{21}$ Wcm^{-2}, $\omega = 1.54$ eV, $E_\mathrm{kin} = 10$ eV and $N = 10^3$. In the left panel $\boldsymbol{\epsilon}'$ is in the scattering plane and in the right panel perpendicular to it.

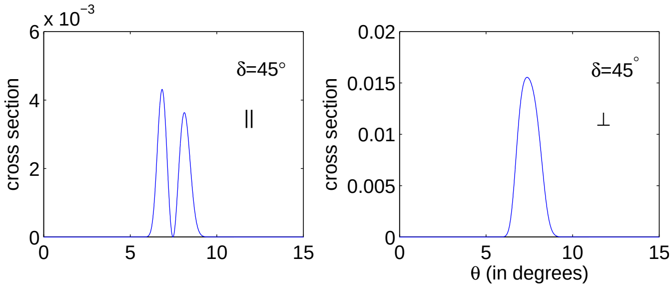

Fig. 2. The same as in Figure 1 but for circular polarization ($\delta = 45°$).

In Figure 2 we present similar spectra for circular polarization of the laser beam ($\delta = 45°$). Here we find very different spectra of the Compton light for $\|$ and for $\perp$ polarization and the emitted radiation appears at larger scattering angles with rather similar cross-section values that are, however, very much smaller than the cross-sections for linear laser polarization and $\|$ polarized Compton light. This confirms our earlier findings that linear laser polarization leads to higher efficiencies of laser-induced nonlinear processes.

2 Electron-atom scattering

2.1 Introduction

The original work on free-free transitions in a powerful laser field is clearly summarized in the overview by Bunkin *et al.* [9]. More elaborate introductions into this field of research can be found in two books [10] and summaries of more recent work are presented in several reviews [11]. With the advent of very powerful laser sources, yielding intensities of 10^{18} Wcm^{-2} and above, it has become important to consider laser-modified and laser-induced processes relativistically [3,12]. Hence, Mott scattering in a powerful, circularly polarized radiation field was reconsidered very recently by Szymanowski *et al.* [13]. This configuration is, however, not so effective and rich in details as in the case of linear polarization, as we were able to show in our earlier semi-relativistic and relativistic work on this problem, neglecting in a first order of approximation the effects of the electron spin [14,15]. Apparently, for linearly polarized laser light the oscillating electron will encounter more often the target atom during the scattering process and therefore the laser-modified collision process will, in the relativistic case, be much more effective and richer in its angular and polarization dependences than for circular laser polarization, as we discussed in our earlier work [15]. It is the purpose of the present investigation to re-analyze all these effects in more detail, in particular for a very powerful, elliptically polarized laser field.

2.2 Relativistic cross-section formula

The required Volkov solution for an electron of initial four-momentum $p_{\rm i}$ and a similar solution for the scattered electron of final four-momentum $p_{\rm f}$ can be easily written down. For Mott scattering in a powerful laser field in the first order Born approximation we have to evaluate the T-matrix element

$$T_{fi} = -{\rm i} \int {\rm d}x \; \overline{\psi}_{p_{\rm f}}(x) \gamma^{\mu} U_{\mu}(x) \psi_{p_{\rm i}}. \tag{7}$$

We can decompose this matrix element into its Fourier components in space and time, using the elliptically polarized electromagnetic plane wave field in the Coulomb gauge given in equation (3). Then we find by straightforward calculation

$$T_{fi} = -{\rm i}\frac{m}{V\sqrt{E_{\rm f}E_{\rm i}}} \sum_N \int {\rm d}t \exp\left[{\rm i}(\overline{E}_{\rm f} - \overline{E}_{\rm i} - N\omega)t\right] \\ \times \int {\rm d}\mathbf{r} U(\mathbf{r}) \exp\left[-{\rm i}(\overline{\mathbf{p}}_{\rm f} - \overline{\mathbf{p}}_{\rm i} - N\omega\mathbf{n})\cdot\mathbf{r}\right] M_N \tag{8}$$

where $U(\mathbf{r})$ is a screened Coulomb potential of charge eZ and screening length ℓ and the nonlinear matrix elements M_N are given by a very lengthy expression, not written down here, which can be evaluated numerically. The Fourier transform $U(\mathbf{q})$ of the screened Coulomb potential can be easily obtained so that we are able to evaluate from equation (8) the transition probabilities per unit time and, finally the differential cross-sections of the non-linear scattering processes of the order N. This yields

$$\frac{{\rm d}\sigma_N^{(s,s')}}{{\rm d}\Omega'} = \frac{(p_{\rm f})_N}{p_{\rm i}} \frac{4(mZ\alpha)^2}{F_N} \frac{|M_N|^2}{[(\overline{\mathbf{p}}_{\rm f} - \overline{\mathbf{p}}_{\rm i} - N\mathbf{k})^2 + \ell^{-2}]^2}. \tag{9}$$

In this equation, the indices s and s' are labelling the spin polarizations of the incoming and outgoing electrons and have the possible values $+$ or $-$. These indices, $\pm$, refer to the spin polarization of an electron in its rest frame, having the values $\pm 1/2$ with respect to the z-axis. The energy conservation relation, that follows from equation (8), reads $\bar{E}_{\rm f} = \bar{E}_{\rm i} + N\omega$ and the evaluation of the phase space leads to the function F_N, given by

$$F_N = 1 - \frac{(d_{\rm f})_N}{(E_{\rm f})_N - (\mathbf{p}_{\rm f})_N\cdot\mathbf{n}} \left(1 - \frac{(E_{\rm f})_N(\mathbf{p}_{\rm f})_N\cdot\mathbf{n}}{(p_{\rm f})_N^2}\right). \tag{10}$$

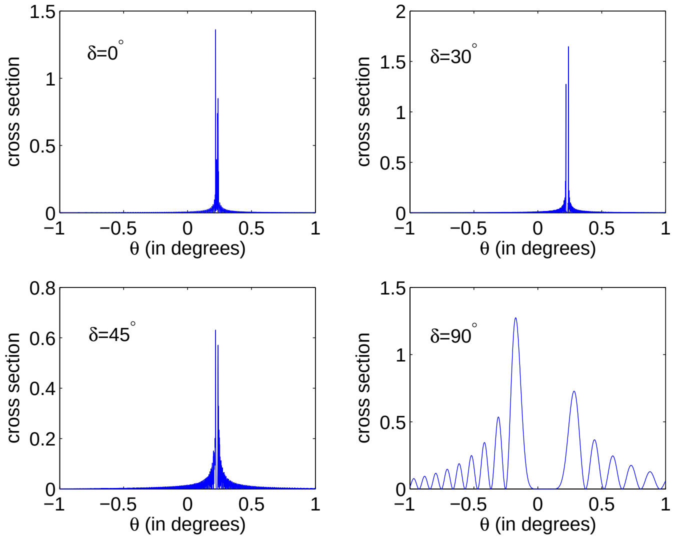

Fig. 3. Cross-sections of Mott scattering in a.u. for $I = 10^{18}$ Wcm^{-2}, $\omega = 1.17$ eV, $E_{\rm kin} = 200$ eV and $N = 10^3$, evaluated for four different helicities.

2.3 Numerical examples

We show cross-section data for Mott scattering at the laser field intensity $I = 10^{18}$ Wcm^{-2} and for $\omega = 1.17$ eV. The number of absorbed photons is $N = 10^3$ and the ingoing electrons have kinetic energy $E_{\rm kin} = 200$ eV. The laser beam and electron beam cross under 90° and determine the scattering plane.

In Figure 3 we show in four panels scattering data in a.u. for the helicities $\delta = 0°$, $\delta = 30°$, $\delta = 45°$ and for $\delta = 90°$. Evidently, under these conditions scattering takes place into very small scattering angles θ and the cross-section values are rather similar for all helicity values δ. Moreover, we recognize a shift of the scattering peaks towards positive values of θ, having its origin in the large laser-induced drift velocity of the electrons. The rather δ-independent values of the cross-sections stem from the fact that at such laser intensities the contributions of the $\mathbf{p}\cdot\mathbf{A}$-term and of the $\mathbf{A}^2$-term in the Volkov solution are of about the same order of magnitude.

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